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MAGNETIC INFLUENCE ON THE STABILITY OF LUMINOUS STELLAR ENVELOPES

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ABSTRACT

Turbulence in the convective outer envelope of a luminous post–main-sequence star probably shreds any magnetic field that is buoyed up from the radiative region below and probably also generates small local magnetic fields. These magnetic structures would consist of well-tangled field lines, but could at most attain a steady state strength based on a near equipartition of their mean energy with the turbulent kinetic energy. Even assuming the maximum possible strength, the magnetic fields turn out to be too weak to affect significantly the dynamical stability of the envelope. This is a consequence as much of the low value of the field strength as of the fact that the field responds hydrodynamically to radial perturbations in the same manner as a gas with an adiabatic exponent of 4/3. Furthermore, fields this small are found to barely affect the radiative stability of the envelope. If the strongest magnetic components could safely reach the atmosphere, however, they might exert a noticeable effect there, or if the whole radiative interior were permeated with a strong magnetic field, very rapid mass loss at the surface might keep the outer layers strongly magnetic at all times before turbulence could break up the field lines. Otherwise, the bulk of the outer envelopes in stars such as yellow hypergiants, luminous blue variables, and hydrogen-poor Wolf-Rayet stars are not expected to be strongly magnetic.

Subject headings: stars: magnetic fields — stars: oscillations — stars: variables: other — stars: Wolf-Rayet — turbulence

1. INTRODUCTION

The envelope of a luminous post–main-sequence star feels a number of forces acting on it besides the inward force of gravity and the combined outward forces exerted by gas pressure and radiation pressure. These other forces are produced by turbulent pressure, axial rotation, mass loss, and magnetic fields. While their effects have been much studied in the case of main-sequence stars, less is known about them in more evolved stars. Of special interest here are luminous stars on the verge of dynamical instability or of radiative instability (de Jager et al. 2001). How do these additional forces affect the stability properties of such stars? The classes of stars that might be relevant, if they are post–main-sequence stars, consist of yellow hypergiants, luminous blue variable stars (LBVs or S Doradus variables), and Wolf-Rayet stars.

Three of these additional forces have been recently studied to answer this question. The outward acceleration of matter due to surface mass loss (the stellar wind) tends to destabilize the stellar envelope, sometimes very strongly, but turbulent pressure and axial rotation turn out to be too feeble to have any significant effect (Stothers 1999a, 2002, 2003a). This leaves us with magnetic fields.

The origin of any magnetic field in a post–main-sequence star may be either primordial or recent. If the field is fossil, it may be a leftover from the main-sequence phase. However, the hydrogen-poor layers that are now exposed at the surface of the star once lay inside the hydrogen-burning convective core and later became part of the intermediate convection zone that developed just above the hydrogen-burning shell. In these strongly convective regions, it is likely that the magnetic field lines became twisted and shredded into small flux tubes by the turbulent motions: a phenomenon demonstrated in numerical simulations of the Sun's convective envelope (Tobias et al. 2001). Therefore, we do not expect to find a large-scale magnetic field in the present envelopes of luminous post–main-sequence stars.

With or without a seed magnetic field (and even without any axial rotation of the star), a small-scale magnetic field can be built up by turbulent motions alone. Batchelor (1950) showed that such spontaneous field generation is possible in an electrically conductive medium if $4\pi\sigma\nu > 1$, where σ is the electrical conductivity and ν is the kinematic viscosity. Since the kinematic viscosity of a stellar envelope is too small to be effective, Kulsrud (1955) suggested that eddy viscosity might work. In fact, eddy viscosity may well explain the large-scale structural features of turbulence itself (Stothers 2000; Canuto 2000). Numerical simulations have confirmed the feasibility of generating magnetic fields by such a turbulent dynamo (Meneguzzi & Pouquet 1989; Cattaneo 1999). The amplitude of the steady state field is difficult to estimate, but Thelen & Cattaneo (2000) have suggested a mean magnetic energy of up to 25% of the turbulent kinetic energy.

Another possible source for the magnetic field is rotational dynamo action at the base of the outer convection zone if the envelope is rotating sufficiently fast, as in the case of the Sun (Parker 1993; Tobias et al. 2001). A toroidal field is generated at the base and floats up toward the surface. Only the strongest component of the field reaches the atmosphere; the rest gets broken up by turbulence and converted into a poloidal field. The magnetic field might also originate from dynamo action in the helium-burning convective core. Although ordinary diffusion processes and rotation-induced meridional circulation are probably too slow to be able to conduct such a deep-seated magnetic field through the radiative intermediate layers, buoyancy forces acting on the flux tubes might lift them much faster (MacGregor & Cassinelli 2003).

The upshot of this brief analysis is that the outer convection zone contains, most likely, a small-scale magnetic field that is strongest in layers in which the turbulence is strongest. The maximum possible field strength would be that of an equipartition field. Turbulence no doubt scrambles the field lines very thoroughly. With these assumptions, our present investigation of the effects of a magnetic field on post–main-sequence stellar envelopes can now proceed to a treatment of ionization-induced dynamical instability in §§ 2 and 3 and of radiative instability in § 4. Our final conclusions are summarized in § 5.

2. DYNAMICAL INSTABILITY

To estimate the maximum likely strength of the magnetic field $\boldsymbol{H}$ in the turbulent layers of the envelope, we assume that the average local magnetic energy exists in equipartition with the local turbulent energy:

$$\frac{\langle H^2 \rangle}{8\pi} = \frac{1}{2}\rho v_{\rm turb}^2. \tag{1}$$

Here ρ is the mass density and $v_{\rm turb}$ is the mean turbulent velocity. A reasonable approximation is to equate $v_{\rm turb}$ with the mean convective velocity derived from standard mixing-length theory (Cox & Giuli 1968; Stothers 2003a). In the nonturbulent layers we set $\langle H^2 \rangle = 0$. The mean squared magnetic field strength is here a spatial average taken over a spherical shell.

If the magnetic field lines are well tangled within the fluid, they will exert stresses that are nearly isotropic. Then the mean radial component is simply $\langle H_r^2 \rangle = \frac{1}{3}\langle H^2 \rangle$. Trasco (1970) has shown that the Lorentz force on the fluid reduces, in this case, to the space derivative of a magnetic pressure, $P_{\rm mag} = \langle H^2 \rangle / 24\pi$. It is assumed that spherical symmetry of the star is preserved.

The equation of motion then becomes

$$\frac{d^2 r}{dt^2} = -\frac{1}{\rho}\frac{d}{dr}\left(P + \frac{\langle H^2 \rangle}{24\pi}\right) - g + f, \tag{2}$$

where $P = P_{\rm gas} + P_{\rm rad}$ represents the thermodynamic pressure, $g = GM(r)/r^2$ is the gravitational acceleration, and f is the mass-loss acceleration. We have neglected the turbulent pressure and have replaced it with the magnetic pressure for two reasons. First, the magnetic field, once built up, tends to weaken the turbulence (Peckover & Weiss 1978; Tayler 1986; Cattaneo & Vainshtein 1991; Cattaneo 1994; Lee et al. 2003). Second, our main purpose is to elucidate and quantify the effect of the magnetic field, without the complicating effects of other forces that could confuse the interpretation.

Under these assumptions, in particular a magnetic field that does not interact further with the turbulence, it follows that a small overall radial displacement of the fluid will carry along with it the magnetic lines of force, so that the magnetic flux is locally conserved. The associated change is (Stothers 1979)

$$\frac{\delta \langle H^2 \rangle}{\langle H^2 \rangle} = \frac{4}{3}\frac{\delta\rho}{\rho}. \tag{3}$$

Thus, the magnetic field behaves as a gas with an adiabatic exponent of 4/3. This result is general and does not depend on how magnetic energy is partitioned with turbulent energy. For simplicity, and with some physical justification (Stothers 2002), we also assume $f \propto g$; therefore, the ratio $\psi = f/g$ is a constant in space. Finally, continuity of matter throughout the star requires

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho. \tag{4}$$

Assuming small radial adiabatic perturbations, as is appropriate for the consideration of dynamical instability (Stothers 1999b), we write, for example, $r = r_0 + \delta r \exp(i\sigma t)$, where σ is a complex pulsation frequency. Adiabaticity requires $\delta P/P_0 = \Gamma_1 \delta\rho/\rho_0$. Linearizing equations (2)–(4) and then dropping the zero subscripts, we find

$$\begin{aligned}
&\frac{d^2}{dr^2}\left(\frac{\delta r}{r}\right) \\
&+\left[\frac{4-V}{r} + \frac{1}{1+4\nu/3\Gamma_1}\left(\frac{C}{r} + \frac{4\nu/3\Gamma_1}{r}\frac{d\ln\nu}{d\ln r}\right)\right]\frac{d}{dr}\left(\frac{\delta r}{r}\right) \\
&+\frac{1}{1+4\nu/3\Gamma_1}\left(\frac{V}{\Gamma_1 r^2}\right) \\
&\times\left[\frac{\sigma^2 r^3}{GM(r)(1-\psi-\mu)} - (3\Gamma_1 - 4) + \frac{3\Gamma_1 C}{V}\right]\frac{\delta r}{r} = 0,
\end{aligned} \tag{5}$$

where $V = -(d\ln P)/(d\ln r)$, $C = (d\ln\Gamma_1)/(d\ln r)$, and

$$\nu = \frac{\langle H^2 \rangle}{24\pi P}, \quad \mu = -\frac{1}{g\rho}\frac{d}{dr}\left(\frac{\langle H^2 \rangle}{24\pi}\right). \tag{6}$$

For dynamical instability, it is necessary and sufficient that $\sigma^2 \leq 0$, where σ is the smallest eigenvalue for which $\delta r/r$ is finite at the stellar surface and 0 at the base of the envelope.

The one-zone model of a stellar envelope with uniform Γ_1, according to equation (5) with $\delta r/r = \text{const}$, becomes dynamically unstable if $\Gamma_1 \leq 4/3$, as long as $1 - \psi - \mu > 0$. This result is identical to the standard criterion derived for nonmagnetic stars, because the magnetic field lines behave as a gas with $\Gamma_1 = 4/3$. This result for the one-zone model has been obtained before (Stothers 1981), although not as a special case of a more general theory such as the one considered here. It has also been obtained even earlier for a uniform-density stellar model (Chandrasekhar & Limber 1954).

It might be thought possible to write for the total pressure $P = P_{\rm gas} + P_{\rm rad} + P_{\rm mag}$ and to incorporate $P_{\rm mag}$, with a corresponding specific magnetic energy $E_{\rm mag}$, into the thermodynamic identity, in order to derive generalized values of Γ_1 and Γ_2. This approach has been used before by Tutukov & Ruben (1974), who assumed $P_{\rm mag} = K\rho^{4/3}$ with constant K, and by Mollikutty et al. (1989), who assumed $P_{\rm mag} = KP_{\rm gas}$ with constant K. Assumptions such as these, however, are too narrow to be applicable to our models (compare Fig. 1). Another possible approach is that of Lydon & Sofia (1995) and Li & Sofia (2001), who assumed that the integral of the specific magnetic energy over the mass of the star is conserved during an adiabatic change. All in all, however, our present, more direct approach seems simplest and least objectionable in theory, as it follows the very general methodology of Chandrasekhar & Fermi (1953) and Chandrasekhar & Limber (1954).

3. APPLICATIONS TO DETAILED MODELS

To assess quantitatively the effects of tangled magnetic fields on realistic stellar envelope models, we consider two very luminous models at the borderline of dynamical instability. These were studied previously to determine the effects of turbulent pressure on their stability (Stothers 2003a). Both models possess $M/M_\odot = 21.6$ and $\log(L/L_\odot) = 5.802$, and convective quantities in them were computed with a ratio of convective mixing length to local pressure scale height, α_P, equal to 1.4. One model represents a yellow hypergiant with

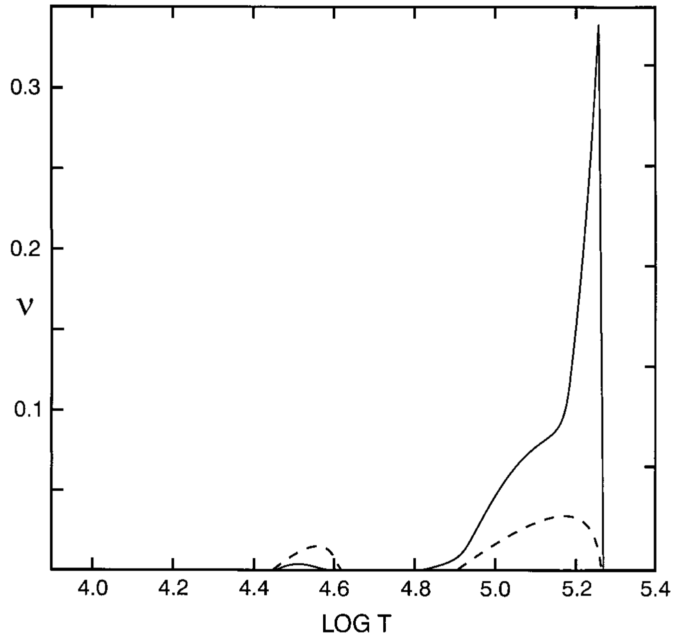

FIG. 1.—Run of the maximum expected ratio of magnetic pressure to thermodynamic pressure (based on an assumed equipartition of magnetic energy and turbulent kinetic energy) through the outer envelope of two post–main-sequence stellar models with $M/M_{\odot} = 21.6$, $\log(L/L_{\odot}) = 5.802$, and $\alpha_P = 1.4$. The yellow hypergiant model has $\log T_e = 3.97$ and $f/g = 0$ (*dashed line*), and the LBV model has $\log T_e = 4.35$ and $f/g = 0.16$ (*solid line*). Both models sit on the borderline of dynamical instability.

$\log T_e = 3.97$ and the other an LBV with $\log T_e = 4.35$. We replace here the turbulent pressure with the magnetic pressure $\langle H^2 \rangle / 24\pi$, calculated from equation (1). Figure 1 shows the runs of the ratio $\nu = \langle H^2 \rangle / 24\pi P$ through the two envelope models.

The magnetic field affects the models in two ways. First, it changes the effective gravity of the equilibrium model through the inclusion of the Lorentz force:

$$\frac{1}{\rho} \frac{dP}{dr} = -g_{\text{eff}} = -g(1 - \psi - \mu). \tag{7}$$

Second, it interacts with the oscillations of the model around the equilibrium state.

In practice, however, the influence of a magnetic field turns out to be negligible. The reason is that dynamical stability is determined primarily by the layers cooler than 1×10^5 K (Stothers 2003a). Figure 1 shows that ν is generally very close to 0 and nowhere exceeds 0.05 in the outer layers, although it may grow large deep inside the iron convection zone ($T \sim 1.5 \times 10^5$ K), where turbulence becomes supersonic for the bluest stars. Since $|\mu| \sim \nu$ for $\nu \ll 1$, μ also is locally small and is even less effective than ν because it changes sign in the helium convection zone ($T \sim 3 \times 10^4$ K), leading to a near cancellation of its small modification of the effective gravity there. Since the oscillation amplitude $\delta r/r$ remains nearly constant above the iron convection zone, the interaction of the magnetic field with the oscillations has essentially no effect on the standard criterion for dynamical instability, as a simple inspection of equation (5) easily reveals.

Thus, the effect of the magnetic field on dynamical instability proves to be even less than the effect of turbulent pressure, because the magnetic field adjusts as a gas with an adiabatic exponent of 4/3, but the turbulent pressure in the outer layers is nearly static, adapting scarcely at all to the oscillations owing to its long timescale (Stothers 2003a). The estimated maximum strength of the magnetic field is found to be very small: less than 10^2 G in the helium convection zone and less than 10^4 G in the iron convection zone.

If, however, surface mass loss happens to be so rapid that the exposed layers display a strong pre-existing interior magnetic field (§ 4), and if this magnetic field increases with depth, it would counter gravity to some extent. Densities would then drop, bringing Γ_1 closer to 4/3. The critically needed rates of mass loss that we calculated previously for the threshold of dynamical instability (Stothers 2002) would accordingly be reduced by a factor of $(1+\nu)^{1/2}(1+\nu/\psi)^{-1/2}$ if ν were spatially constant. However, we do not actually anticipate the presence of large interior magnetic fields.

4. RADIATIVE INSTABILITY

Although, formally, most of the outer envelope of a very luminous star is convectively unstable, the densities are so low that energy is transported almost entirely by radiative processes (except in the iron convection zone, which is nearly adiabatic). This circumstance simplifies matters considerably.

The equation of radiative transfer is given by

$$\frac{dP_{\text{rad}}}{dr} = -\frac{\kappa \rho L}{4\pi c r^2}. \tag{8}$$

Dividing equation (8) into equation (7) with $M(r) = M$ and integrating from the surface down to a layer at r gives

$$1 - \beta = \frac{\langle \kappa \rangle L}{4\pi c G M (1 - \psi - \mu)}. \tag{9}$$

Here $\beta = P_{\text{gas}}/P$ and

$$\frac{1}{\langle \kappa \rangle} = \frac{1}{(1 - \psi - \mu) P_{\text{rad}}} \int_R^r \frac{1 - \psi - \mu}{\kappa} \frac{dP_{\text{rad}}}{dr} \, dr. \tag{10}$$

Since, physically, β cannot be less than 0, there exists an upper limit to the luminosity of the stellar envelope that preserves radiative stability:

$$L_{\text{E}} = \frac{4\pi c G M (1 - \psi - \mu)}{\langle \kappa \rangle}. \tag{11}$$

This upper limit represents a generalization of the original Eddington luminosity, since it incorporates the mass-loss acceleration and the effect of magnetic fields, which Eddington (1921) ignored.

In practice, little change from our earlier models without magnetic fields is incurred, because μ is usually very small. Only in the iron convection zone of the brightest and hottest models, in which turbulence becomes supersonic, could a strong magnetic field be generated (Fig. 1). In that case, both turbulence and the magnetic field would tend to promote radiative instability.

Another possibility is that a strong magnetic field lies embedded in the underlying radiative zone. If the rate of mass loss is sufficiently high, the overlying layers may get stripped off in a time short compared to one, or a few, convective overturning times. Since in this case the outer layers would not have time either to generate a new magnetic field or to shred an old one, the instantaneous envelope would then maintain the strong pre-existing magnetic field everywhere. A uniform magnetic field would have no effect, because for such a field $\mu = 0$. Assuming, however, that ν is constant in space, we would have $1 - \psi - \mu = (1 - \psi)/(1 + \nu)$; the reduction in radiative stability might be considerable for large enough ν. In addition, if full-scale turbulence itself does not have sufficient time to develop, the zone with a large iron opacity bump might become nearly radiative, in which case the opacity bump alone could cause radiative instability (Kato & Iben 1992). These uncertainties concerning the iron convection zone of course make it difficult to project a realistic value for the rate of mass loss. However, the *minimum* rate of mass loss needed for radiative instability is probably safely predictable to within an order of magnitude (Stothers 2003b).

5. CONCLUSION

The present study of magnetic fields in the outer envelopes of luminous post–main-sequence stars suggests that any magnetic fields sequestered there are probably small-scale, weak, and under the control of turbulence. In that case, they should have very little influence on the dynamical stability and radiative stability of the envelope. Our previous conclusions for yellow hypergiants, most LBVs, and most hydrogen-poor Wolf-Rayet stars, based on nonmagnetic stellar models, therefore remain unchanged.

A somewhat different implication, however, follows for the brightest and hottest of these stars. If supersonic turbulence manages to build up a strong magnetic field deep inside the iron convection zone (or in a rotating tachocline at the interface with the underlying radiative zone), the strongest component of the magnetic field might be buoyed up to the surface, where it would affect the structure and stability of the atmosphere. Previously, a magnetic field was not considered to be an important factor for the atmosphere (de Jager et al. 2001). Much, of course, depends on the rate of stellar-wind mass loss. If the rate were very high, there would not be time enough for turbulence to generate and convey upward the magnetic flux tubes. On the other hand, if the radiative interior already stored a significant magnetic field, the rapid exposure of these layers by surface mass loss would ensure that the whole outer envelope would remain constantly permeated with a strong magnetic field. Observations of the stellar wind with respect to its overall shape and also with respect to the shapes of its spectral lines can possibly reveal useful information about the interior magnetic field. If the bipolar shapes of some LBV nebulae are not due to axial rotation or to duplicity of the underlying star, then perhaps a strong magnetic field may be responsible for it.

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